Some Comments on W. Horowitz, "A Late Babylonian Tablet with Concentric Circles"

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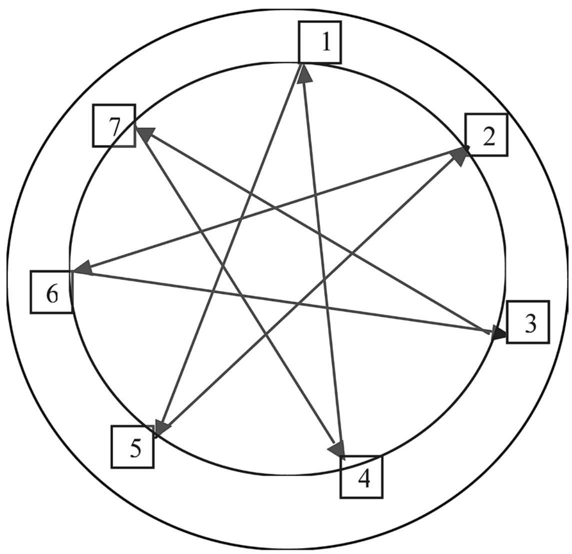
Tablet CBS 1766 from the University of Pennsylvania Museum has been analyzed in detail by W. Horowitz.[1] The tablet contains a diagram of a seven-pointed star inscribed in the inner of two concentric circles along with a numerical table composed of four columns of seven entries and four more columns with only the initial entries.



The points of the star are labeled with the cuneiform signs for the numbers 1 to 7 and the line segments forming the star connect the points with numbers which are adjacent in columns A and B. Horowitz notes that, "it seems fair to assume that the table corresponds in some way to the diagram with its seven pointed star." In his interpretation of the table, Horowitz considers the sets of numbers in row-adjacent number-pairs in columns A-B, C-D, E-F, and G-H in relation to the concentric circles in which the star is inscribed. He interprets the tables as describing a shift in the labeling of the star given by a rotation of one circle inside the other.

1. W. Horowitz, "A Late Babylonian Tablet with Concentric Circles from the University Museum (CBS 1766)," *JANES* 30 (2006), 37–53, quote on p. 41.

Based on a different reading of the signs at the points of the star, C. Waerzeggers and R. Siebes have interpreted the tablet as a tuning chart for a seven-stringed lyre.[2] Their alternative reading has been confirmed in part by a second look at the tablet.[3] L. Crickmore has given a detailed analysis of the tuning procedure.[4] While preparing this article, I received a preprint from Joran Friberg discussing, among other examples, tablet CBS 1766.[5] His mathematical explanation of the star and two other geometric figures which are encoded in the tables is similar to ours. In addition, he also explains their relationship to the tuning procedure. Finally, he proposes filling in the empty columns with pairs that describe the sides in two inscribed geometric figures, the 7-sided regular polygon (heptagon) and the 7-pointed star with 77 degree angles.[6] I shall suggest a different way of filling in the missing columns, which describes the same geometry but also preserves a unifying pattern.

I would offer a partial explanation of the tablet based on a simple mathematical fact connecting the arithmetic of the table and the geometry of the star, and on that basis to make a connection with astronomical tables, namely:

> *The numbers in all the columns come from the same arithmetic sequence modulo 7 (except perhaps for the last entry in columns A and C), and there are many examples of such sequences which appear in the Babylonian astronomical tables.*

In fact, in some of the astronomical tables one observes a similar divergence from a strict arithmetic progression in the last entry. The arithmetic sequences (or progressions) in the columns are given by successively adding 4 and, if necessary, reducing modulo 7; that is, if the number given by adding 4 is greater than 7, it is reduced by subtracting 7. Each column follows the same pattern but starts with a different initial value. In columns A and C the first 6 entries form the arithmetic sequence and the last entry adds 5 instead of 4. The entries in columns A through D are shown in the following table.

A	B	C	D
2	6	1	7
6 = 2+4	3 = 6+4 mod 7	5 = 1+4	4 = 7+4
3 = 6+4 mod 7	7 = 3+4	2 = 5+4 mod 7	1 = 4+4 mod 7
7 = 3+4	4 = 7+4 mod 7	6 = 2+4	5 = 1+4
4 = 7+4 mod 7	1 = 4+4 mod 7	3 = 6+4 mod 7	2 = 5+4 mod 7
1 = 4+4 mod 7	5 = 1+4	7 = 3+4	6 = 2+4
6 ?	2 = 5+4 mod 7	5?	3 = 6+4 mod 7

2. C. Waerzeggers and R. Siebes, "An Alternative Interpretation of the Seven-pointed Star on CBS 1766," *NABU* 2007/2, 43–45. See also J. Smith and A. Kilmer, "Laying the Rough, Testing the Fine," in E. Hickman and R. Eichmann, eds., *Studien zur Musikarchäologie I, Orient-Archäologie* 6 (2000), 127–40.

3. W. Horowitz and S. Shnider, "Return to CBS 1766," *NABU*, 2009/1, 7–9.

4. L. Crickmore, "A Musical and Mathematical Context for CBS 1766," *Music Theory Spectrum* 30 (2008), 327–59.

5. J. Friberg, "Polygons, Star Figures, and Tuning Algorithms in Mesopotamian Cuneiform Texts," preprint. See also idem, "Seven-sided Star Figures and Tuning Algorithms in Mesopotamian, Greek, and Islamic Texts," *AfO* 52 (2011), 121–55.

6. Friberg, "Polygons," 12–13.

To see the relationship to the geometry of the star, note that if seven equidistant points are laid out on the circumference of a circle and are then connected in sequence by straight lines (chords) according to the sequences in any of the columns, the seven-pointed star is traced out almost completely. The last segment is lacking unless we continue the sequence an additional step. The same star is traced out if we connect row-adjacent numbers in columns A and B, closing the star by connecting point 5 to point 2 according to the sequence in column B. If we proceed to connect the points labeled by row-adjacent numbers in the first six rows of columns B and C (numerical difference 2 mod 7) we get six of the line segments making another 7-pointed star, with larger vertex angles than the star in the tablet (approximately 77 degrees as opposed to the approximately 26 degrees in the tablet's star). If we connect the points labeled by row-adjacent numbers in columns C and D with numerical difference 1 or $-6 = 1 \bmod 7$, we get 6 of the sides of an inscribed regular 7-sided polygon. According to this interpretation, the incomplete columns should be filled in with the basic sequence beginning with the number at the top of the column. Then the pairs of numbers in adjacent columns will have a fixed difference, the corresponding line segments will form one of the regular figures mentioned above, and the *basic sequence will be repeated in each column*. This is an alternative to Friberg's completion of the table.

The explanation in mathematical terms is straightforward. It is an elementary fact that since 7 is prime, any arithmetic sequence modulo 7 with non-zero increment will go through all the numbers 1 to 7 exactly once before repeating. By contrast, consider the numbers modulo 6 (not prime). An arithmetic sequence with an increment of 2 will cover only half the numbers modulo 6. The geometric figure corresponding to the sequence will be an equilateral triangle; choosing a new starting point not in the first sequence will generate another equilateral triangle, and the two together will form a Star of David.

From a mathematical perspective, the table translates the components of a very symmetric geometric configuration into arithmetic relations and, conversely, from arithmetic relations to a geometric figure. A similar process of transforming geometry into arithmetic has been observed elsewhere in Babylonian mathematics, as, for example, in the Plimpton tablet, in which the "Law of Pythagoras" for a right triangle becomes a list of Pythagorean triples, describing right triangles with sides of integer length.

There is another context for understanding the tables in CBS 1766, namely, the frequent occurrence of modular arithmetic progressions in Babylonian astronomical tables. Some typical examples of arithmetic sequences in astronomical tables are discussed in an article by John Britton.[7] Take for example the Uruk Scheme. As Britton notes on page 43, the table appearing in Figure 7 is based on a very simple arithmetic progression modulo 30 coming from an epact (excess of 1 year over 12 months) of 11 tithis ($t = 1/30$ of a month) calculated over 19 years. This table also shows an increment of 12 t (instead of 11 t) in the last year. See also the tables in Britton's Appendix C. This pattern is exactly the same as the one we have observed in the tablet CBS 1766: that is, an arithmetic progression, calculated modulo 7 in CBS1766, with an

7. J. Britton, "Treatment of Annual Phenomena in Cuneiform Sources," in J. Steele and A. Imhausen, eds., *Under One Sky* (Münster, 2002), 21–78.

increment of 4, except in the last line of columns A and C, where the increment is increased by one. Similar tables of arithmetic progressions in modular arithmetic appear elsewhere in Britton's article: Figure B3, where the increment is 96, and the entries represent degrees (hence modulo 360); and again in Figure B4 where the increment is 11 modulo 30. In my opinion the similarity is too great to be purely coincidental.

Another example of an arithmetic progression (though not modular) in early astronomical tables is Tablet 14 in the *Enūma Anu Enlil* omen series.[8] The tables describe the intervals of time between sunset/moonset, moonrise/sunrise, sunset/moonrise for the days of an ideal month, or for selected days of the months. For example, Table A records the interval of time between sunset and moonset for each day of the first half of an ideal month at the equinox, and the interval from moonrise to sunrise and sunset to moonrise for each day of the second half of the month. The list begins with a geometrical progression for the first 5 days (doubling) and is followed by an arithmetic progression increasing by 12 *Uš* for the next 10 days. Then, as in a linear zigzag function, the progression begins decreasing with the same interval for the next 10 days and becomes a decreasing geometric progression (halving) until day 30. Other noteworthy examples of modular arithmetic progressions appear in the Neo-Babylonian tablets BM 96258 and BM 96293.[9]

An interesting possibility to consider is that that these different interpretations reflect a way in which the Babylonian scribe who created the tablet connected the three areas of study—mathematics, astronomy, and music. Francesca Rochberg has studied the cultural context of science in late Babylonia. She observes that the texts in the ACT series "represent the highly refined and thoroughly mathematical treatment of astronomical phenomena in tabular form, and are now known to be part of the responsibilities of the *tupšar Enūma Anu Enlil* . . . In light of CT 49, 144, then, as well as in a small number of other contemporary texts in which the protagonists are named as scribes, the scribes of *Enūma Anu Enlil* may be said to have been members of the priesthood and had their hands in divination, observational astronomy, and exact science (i.e. mathematical astronomy) simultaneously."[10] Since music was part of the temple service, the observation of mathematical patterns in music was certainly within their purview.

A possible further connection with astronomy appears in the line of text at the bottom of the diagram. Horowitz translates the expression *ṣi-im-da-tum zi-qí-pu* as *pairs of altitudes*, and goes on to explain: "*zi-qí-pu* brings to mind *ziqpu* from the same root [*zaqāpu*, "to erect, to point upward," "to impale"] which has the meaning of "height, altitude" as a mathematical term . . . If this is what is meant by *ziqīpu* in CBS 1766, the circles in the diagram can be compared with the circle on a recently published *ziqpu*-star planisphere from Neo-Babylonian period Sippar."[11]

8. H. Hunger and D. Pingree, *Astral Sciences in Mesopotamia* (Leiden, 1999), 44–50.

9. L. Brack-Bernsen and J. Steele, "Babylonian Mathemagics: Two Mathematical Astronomical-Astrological Texts," in C. Burnett, J. Hogendijk, K. Plofker, M. Yano, eds., *Studies in the History of the Exact Sciences in Honor of David Pingree* (Leiden, 2004), 95–125.

10. F. Rochberg, "The Cultural Locus of Astronomy in Late Babylonia," in H. Galter, *Die Rolle der Astronomie in den Kulturen Mesopotamiens* (Graz, 1993), 31–45, quote taken from pp. 42–43.

11. Horowitz, "A Late Babylonian Tablet," 50.

In the *CAD*, one finds a number of different meanings for *ziqpu*, "shoot" (of a tree), "stake, pole, shaft, blade (of a weapon)," "height, altitude (as a math. term)," and "culmination point (zenith), culminating constellation or star." However, the mathematical meaning "altitude" does not appear as an entry for *ziqīpu,* only for *ziqpu.* The word *ziqīpu* seems to be used only for the "impaling stake," hardly suitable to our context, unless the sides of the star are viewed as stakes perhaps pointing upward. The limited meaning of the term *ziqīpu* makes it reasonable to conjecture that the scribe intended *ziqpu,* suggesting a connection with the *ziqpu* stars and astronomy.

Horowitz goes on to compare the tablet with several other tablets: the Babylonian Map of the World (BM 92687), the *ziqpu* star tablet (AO 6478 = *TCL* 6, 21), and the circular star chart with a twelve-pointed star (*TCL* 6, 13); but he concludes that in spite of the similarities, "there is not one piece of direct evidence tying our text to stars, nor to any other astronomical phenomena." I think that the similarity to the modular arithmetic of the astronomical tables gives at least one more piece of indirect evidence.

In conclusion, this simple tablet may be hinting at the connection between mathematics, astronomy, and music as well as showing the more obvious connection between arithmetical tables and geometric figures. This idea will be developed further in an article currently in preparation with Wayne Horowitz.